

the relationship $Bi_1 = f(Fo_1)$ is unusual for relatively "thin" bodies (Fig. 4). In this case, an extremum is present on the curve $Bi_1 = f(Fo_1)$, whose position shifts toward larger Fo_1 as the dimension of the interacting plates increases. This feature is due to α and, consequently to, Bi_1 . It is apparently explained by "reflection" of the thermal fluxes from an insulated surface of the plate. This effect becomes particularly noticeable for thin plates, as well as for plates with high thermal conductivity.

On the whole the weak dependence of the Biot number on time in almost the entire range of the thermal stabilization process is notable. Despite the fact that strong radiative interaction is clearly defined in the initial stage of thermal stabilization (see Fig. 2), the kinetics of change in the Biot number are in the nature of a sluggish and prolonged process. Here, the rate of decrease in Bi_1 is less in the initial stage than in the later stage.

The above discussion explains to a certain degree why the widely used quasi-stationary methods of calculating nonstationary heat transfer result in satisfactory agreement with experiment.

At the same time the preliminary results presented above provide a basis for performing more detailed

determinations of $\alpha(\tau)$ when there is a distinct thermal interaction.

NOTATION

α_i denotes the thermal diffusivity coefficients; τ is the time; E_i denotes the resulting radiation densities, $i = 1, 2$; $\sigma_{12} = \sigma_{21}$ are the resulting emissivities of the plate; λ_i denotes the thermal-conductivity coefficients for the plate; R_i denotes the plate thickness; T_i is the initial plate temperature; ξ is a dimensionless coordinate; θ_i is the dimensionless temperature; $\varphi(\tau)$ is the dimensionless resulting radiation density; $Fo_1 = (\lambda_1/c_1\rho_1)(\tau/R^2)$ is the Fourier number; $Bi_1 = \alpha_1 R_1/\lambda_1$ is the Biot number.

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14 September 1966

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THERMAL CONDITIONS FOR PRODUCING ARTICLES FROM A MELT

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Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 2, pp. 255-258, 1967

UDC 621.036:669

The temperature distribution is indicated through directly measurable values from consideration of convection equations for the steady-state process. The impossibility of stabilization of the interface when the extraction rate changes is shown.

A. V. Stepanov has proposed a method for producing articles directly from a melt [1].

Various articles made from a number of materials are now being produced with this method [2, 3]. Which basically is as follows. The melt column is given the desired shape (see figure), and finished article is obtained by crystallization of the column. The importance of calculation of the position of the crystallization front is obvious here. This problem is also of interest for the Czochralski method.

The thermal conditions of the process for a specified transition-boundary position are calculated in this article in an approximation of a one-dimensional thermal problem with convection.

The cooling schemes can vary [3]. We assume that heat transfer occurs only due to internal thermal conductivity. The coordinate system is shown in the figure.

We ignore change in the physical characteristics of the material on either side of the phase interface.

The equations of the problem are [4]

$$\frac{\partial \theta_1}{\partial t} + u \frac{\partial \theta_1}{\partial x} - a^2 \frac{\partial^2 \theta_1}{\partial x^2} = 0, \quad 0 \leq x \leq X(t), \quad (1)$$

$$\frac{\partial \theta_2}{\partial t} + u \frac{\partial \theta_2}{\partial x} - a^2 \frac{\partial^2 \theta_2}{\partial x^2} = 0, \quad X(t) \leq x. \quad (2)$$

At the boundary, we have

$$\theta_1(X, t) = \theta_2(X, t) = T_0, \quad (3)$$

$$K \left(\frac{\partial \theta_2}{\partial x} - \frac{\partial \theta_1}{\partial x} \right) \Big|_{x=X} = Lu\rho + L\rho \frac{dX}{dt}, \quad (4)$$

$$\theta_1(0, t) = T_m.$$

The simplest case of the stabilization problem consists, with steady-state extraction and therefore with a fixed interface, of maintaining the position of

the boundary by proper selection of the temperature in the cooling zone when the extraction rate varies. However, it is easy to see that this requirement is not feasible in such a rigid form.

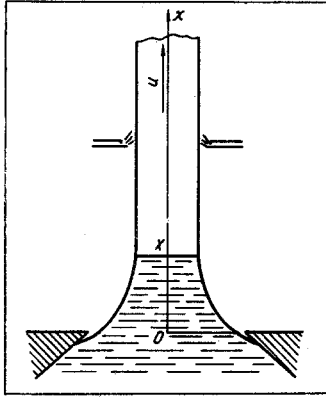


Diagram of production from a melt.

Let us make the substitution $\vartheta_2^* = \vartheta_2 - T_0$, $y = x - X$. By the substitution

$$\vartheta_2^* = \exp\left(\frac{u_1}{2a^2}x - \frac{u_1^2}{4a^2}t\right) s(x, t)$$

we transform Eq. (2) into the heat-conduction equation

$$\frac{\partial s}{\partial t} = a^2 \frac{\partial^2 s}{\partial y^2}$$

(u_1 is the new extraction rate) with the additional conditions

$$s(y, t)|_{y=0} = 0, \quad \frac{\partial s}{\partial y} \Big|_{y=0} = \psi(t).$$

Here $\psi(t)$ is a certain known function that increases in t not more rapidly than $\exp(u_1^2 t/4a^2)$. The initial conditions are unimportant for us now, and we let $s(x, 0) \equiv 0$.

We write the Laplace-transform equation for $s(x, t)$

$$\frac{d^2 \bar{s}}{dy^2} - a^2 p \bar{s} = 0.$$

With our boundary condition, its solution will be

$$\bar{s} = \frac{\bar{\psi}(p)}{a \sqrt{p}} \operatorname{sh} ya \sqrt{p}.$$

Inasmuch as $\bar{s}(p)$ does not approach zero as $|p| \rightarrow \infty$, we see that for $s(p)$ a preimage which increases less rapidly than $\exp(u_1^2 t/4a^2)$ does not exist.

Therefore, rigid stabilization is impossible. This result indicates the appropriateness of our introducing, for stabilization purposes, feedback between the temperature in the cooling zone and the interface position.

The temperature distribution in the steady state as a function of the interface position is given by the equations

$$a^2 \frac{\partial^2 \vartheta_i}{\partial x^2} - u \frac{\partial \vartheta_i}{\partial x} = 0 \quad (i = 1, 2)$$

with boundary conditions (3), (4), and $dX/dt \equiv 0$. The result from solution of these equations is

$$\begin{aligned} \vartheta_1 &= T_m - \frac{T_0 - T_m}{\exp\left(\frac{u}{a^2}X\right) - 1} + \\ &+ \frac{T_0 - T_m}{\exp\left(\frac{u}{a^2}X\right) - 1} \exp\left(\frac{u}{a^2}x\right), \quad (5) \\ \vartheta_2 &= T_0 - \left(\frac{L\rho a^2}{K} \exp\left(-\frac{uX}{a^2}\right) + \right. \\ &+ \left. \frac{T_0 - T_m}{\exp\left(\frac{u}{a^2}X\right) - 1}\right) \exp\left(\frac{uX}{a^2}\right) + \\ &+ \left(\frac{L\rho a^2}{K} \exp\left(-\frac{u}{a^2}X\right) + \right. \\ &+ \left. \frac{T_0 - T_m}{\exp\left(\frac{u}{a^2}X\right) - 1}\right) \exp\left(\frac{ux}{a^2}\right). \quad (6) \end{aligned}$$

As is apparent, the latter equation relates the melt temperature, as well as the height and temperature of the crystallization front, to the temperature of any point in the solid phase up to the cooling zone.

A preliminary check of the applicability of this equation was made for extraction with forced air cooling of a strip of technical-grade pure aluminum. The temperature at the base of the melt column and in the

Result of Experimental Check of Formula (6)

u , m/sec	T_m , °C	T_x , °C	T_0 , °C	X_{meas} , m	X_{calc} , m	L , kJ/kg	K , W/m·deg	a^2 , m ² /sec	ρ , kg/m ³
$100 \cdot 10^{-3}$	673	588	645	$1.87 \cdot 10^{-3}$	$2.00 \cdot 10^{-3}$				
$0.70 \cdot 10^{-3}$	670	615	652	$1.74 \cdot 10^{-3}$	$1.76 \cdot 10^{-3}$	394	238	$8.9 \cdot 10^{-5}$	$2.5 \cdot 10^3$
$0.45 \cdot 10^{-3}$	689	599	645	$2.35 \cdot 10^{-3}$	$2.50 \cdot 10^{-3}$				

crystallization zone was measured with a microthermocouple. Then the thermocouple was frozen into the strip and the temperature distribution of the solid phase during crystallization was recorded. Measurement of the phase-transition temperature is necessary, due to the presence of supercooling, which differs for various crystallization rates.

The results are given in the table.

We note that Eq. (6) can be used to determine supercooling, and this is of considerable interest.

NOTATION

u is the extraction rate; T_m is the melt temperature; T_0 is the phase-transition temperature; ϑ_1 is the liquid phase temperature; ϑ_2 is the solid phase temperature, X is the interface coordinate; K is the thermal conductivity coefficient; a^2 is the thermal diffusivity coefficient; L is the specific heat of fusion; ρ is the den-

sity; X_{calc} is the calculated value of the interface coordinate; X_{meas} is the measured value of the interface coordinate.

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15 July 1966

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RETARDATION OF PHASE TRANSITION NEAR THE CRITICAL POINT

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Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 2, pp. 259-263, 1967

UDC 536.753

The nonequilibrium-thermodynamics method is used to examine the kinetics of the liquid-vapor phase transition of a pure substance. It is shown that there is a sharp increase in the pressure-relaxation time in an isothermic system near the critical point.

For a phase transition of the first kind to take place at a finite rate it is necessary to change the conditions for phase equilibrium, for example, to change the pressure while we hold the temperature constant or to maintain a certain temperature difference between phases. However, this situation is often not apparent because heat transfer and hydrodynamic or diffusion processes associated with phase transition predominate.

It has long been known [1] that at low temperatures the relationship between the evaporation rate and the pressure difference $p_s - p$ is given by

$$i = \alpha (2\pi mkT)^{-\frac{1}{2}} (p_s - p). \tag{1}$$

In this case, the evaporation rate is low because of low vapor density, and heat transfer is not a limiting factor. Retardation of phase transition near the critical point is due to some other factor. Here the evaporation rate drops because the difference between coexisting phases becomes negligible.

S. L. Rivkin et al. [2, 3] observed the protracted change (up to 8-10 hours) in water pressure in a two-phase region under isothermal conditions when $T_{Cr} -$

$- T \approx 1-2^\circ \text{C}$. If we do not expect equilibrium to be established, the condensation lines on the p - and v -diagrams will be inclined. The authors of [3] note that this slope is not caused by impurities.

Consider a one-component isolated system consisting of two isotropic coexisting phases. In the general case, we consider that the phase temperatures and pressures differ and are not equal to their values in an equilibrium system. We assume that internal equilibrium in the phases is established much more rapidly than equilibrium between phases, i. e., a quasi-steady state exists for the discontinuous system. We apply the fundamental equation of thermodynamics to each phase:

$$TdS = dU + pdV - \mu dM. \tag{2}$$

With (2) and the conditions that the mass, volume, and internal energy of the entire system be constant, we find the rate of increase in entropy by a direct method:

$$\dot{S} = \left(\frac{1}{T''} - \frac{1}{T'} \right) \dot{U} - \left(\frac{\mu''}{T''} - \frac{\mu'}{T'} \right) \dot{M}''.$$

Therefore, the mass and energy flows have the form

$$J_1 \equiv \dot{M}'' = -L_{11} \left(\frac{\mu''}{T''} - \frac{\mu'}{T'} \right) + L_{12} \left(\frac{1}{T''} - \frac{1}{T'} \right), \tag{3}$$